Eur. Phys. J. D **36**, 129–132 (2005)
DOI: 10.1140/epjd/e2005-00192-7

THE EUROPEAN PHYSICAL JOURNAL D

The classical nature of signal velocity not greater than c

S.-Y. Zhu[1,2], L.-G. Wang[1,2,a], N.-H. Liu[2,3], and Y.-P. Yang[2,4]

[1] Department of Physics, Zhejiang University, Hangzhou, 310027, P.R. China
[2] Department of Physics, Hong Kong Baptist University, Kowloon Tong, Hong Kong, P.R. China
[3] Department of Physics, Nanchang University, Nanchang 330047, P.R. China
[4] Department of Physics, Tongji University, Shanghai 200092, P.R. China

Received 25 June 2004 / Received in final form 31 January 2005
Published online 26 July 2005 –

Abstract. A classical model is presented to investigate the signal-to-noise ratio (SNR) for pulse propagation through an anomalous dispersive medium. By comparing the SNRs of the pulse passing through the medium and through the vacuum, we show that the problem of the signal velocity not faster than c can be considered within the scope of classical theory. It is shown that, by including the classical fluctuations, the signal velocity is not larger than c.

PACS. 03.65.Sq Semiclassical theories and applications – 42.50.Lc Quantum fluctuations, quantum noise, and quantum jumps – 42.25.Bs Wave propagation, transmission and absorption

The phenomenon that the group velocity v_g is larger than the vacuum light speed c or even becomes negative has been extensively investigated both theoretically and experimentally [1–7]. Due to the recent experimental verification of superluminal pulse propagation [3], the problem of whether a signal can propagate at a velocity faster than c or not has become a hot topic. Although it is claimed that the signal velocity can be larger than c [6–9], most physicists believe it cannot [1,4,10–13]. Initially, Sommerfeld and Brillouin argued that a signal should be defined by a finite pulse with a front and an end, and they showed that the propagation speed of the front can never exceed c. Chiao and his colleague [4,10] also stated that a signal is related to the front or the discontinuous point of a pulse, and the front velocity must be less than or equal to c, thus there is no superluminal signal. Experimental measurements on the frontal velocity with causal behavior have been achieved both electronically [10] and optically [11].

It has been noted that any investigation into whether information can travel superluminally has to take into account noise [14]. Recently, the propagation of a smooth pulse in an anomalous gain medium was investigated within the framework of quantum mechanics [13,15,16]. The quantum fluctuations of the medium were considered. From the operational definition of a signal velocity based on the optical signal-to-noise ratio, they concluded that "quantum noise associated with the amplifying medium acts in effect to retard the observed signal", and "quantum fluctuations limit the signal velocity to values less than c" [13].

Since the superluminal group velocity is a result of classical interference between different frequency components [17–20], we present a classical derivation for the SNR of a pulse propagating through anomalous dispersive media. It is shown that the signal velocity in the medium cannot be faster than c without taking into account the quantum fluctuations. The classical fluctuation prohibits superluminal transfer of a signal.

Consider a pulse propagating through a linear medium of length L. In the linear medium, according to the linear superposition principle, the field can be written as a Fourier integral, $E(z,t) = \int E(z,\omega)\, e^{i(\omega-\omega_0)t} d\omega$, with ω_0 the central frequency of the pulse. At the exit, the light field can be expressed as

$$E(L,\omega) = T(\omega, L)E(0,\omega) + E_n(L,\omega) \tag{1}$$

where $E(0,\omega)$ is the spectrum of the input field which is determined by the incident pulse (signal source), and $E_n(L,\omega)$ is the spectrum of the medium-introduced noise field, $T(\omega,L) = e^{i\omega n(\omega)L/c}$ is the complex transmission coefficient, where $n(\omega)$ is the complex index of the medium.

The integrated total energy at the exit is given by

$$S(L,t) = \int_{-\infty}^{t} dt_1 E(L,t_1)E^*(L,t_1). \tag{2}$$

When the input is zero, the energy at the output port comes from the background noise of the medium, which should be subtracted from the detection by adjusting the

[a] e-mail: sxwlg@yahoo.com.cn

reference point of the detector. Therefore, the effective detected energy is the difference of the total energy and the noise energy:

$$\langle S(L,t)\rangle - \langle S(L,t)\rangle_0 = \int_{-\infty}^{t} dt_1 \,\langle E(L,t_1)E^*(L,t_1)\rangle - \int_{-\infty}^{t} dt_1 \,\langle E(L,t_1)E^*(L,t_1)\rangle_0$$
$$= \frac{1}{2\pi}\int_{-\infty}^{t} dt_1 \int\int d\omega_1 d\omega_2 \; e^{-i(\omega_1-\omega_2)t_1} T(\omega_1,L) \times T^*(\omega_2,L)\,\langle E(0,\omega_1)E^*(0,\omega_2)\rangle \quad (3)$$

where $\langle\;\rangle$ means taking the statistical averages, and $\langle\;\rangle_0$ is for the average without input. We have assumed that there is no correlation between the input field and the noise field. The energy variance of the output field is

$$\left\langle S^2(L,t)\right\rangle - \langle S(L,t)\rangle^2 = \int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2 \,\langle E(L,t_1)E^*(L,t_1)E(L,t_2)E^*(L,t_2)\rangle - \left[\int_{-\infty}^{t} dt_1 \,\langle E(L,t_1)E^*(L,t_1)\rangle\right]^2$$
$$= \frac{1}{(2\pi)^2}\int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2 \times \iiiint d\omega_1 d\omega_2 d\omega_3 d\omega_4 e^{-i(\omega_1 t_1-\omega_2 t_1)} e^{-i(\omega_3 t_2-\omega_4 t_2)}$$
$$\times\{T(\omega_1,L)T^*(\omega_2,L)T(\omega_3,L)T^*(\omega_4,L)\langle\Delta^2\sigma_i\rangle + \langle\Delta^2\sigma_n\rangle + T(\omega_1,L)T^*(\omega_4,L) \times\left\langle E(0,\omega_1)E_n^*(L,\omega_2)E_n(L,\omega_3)E^*(0,\omega_4)\right\rangle + T^*(\omega_2,L)T(\omega_3,L) \times\left\langle E_n(L,\omega_1)E^*(0,\omega_2)E(0,\omega_3)E_n^*(L,\omega_4)\right\rangle\} \quad (4)$$

where

$$\left\langle\Delta^2\sigma_i\right\rangle = \langle E(0,\omega_1)E^*(0,\omega_2)E(0,\omega_3)E^*(0,\omega_4)\rangle - \langle E(0,\omega_1)E^*(0,\omega_2)\rangle\,\langle E(0,\omega_3)E^*(0,\omega_4)\rangle$$

is the fluctuation of the input pulse, and

$$\left\langle\Delta^2\sigma_n\right\rangle = \langle E_n(L,\omega_1)E_n^*(L,\omega_2)E_n(L,\omega_3)E_n^*(L,\omega_4)\rangle - \langle E_n(L,\omega_1)E_n^*(L,\omega_2)\rangle\,\langle E_n(L,\omega_3)E_n^*(L,\omega_4)\rangle$$

is the fluctuation of the background noise. Similarly, the energy variance at the exit end without an input is,

$$\left\langle S^2(L,t)\right\rangle_0 - \langle S(L,t)\rangle_0^2 = \int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2 \,\langle E(L,t_1)E^*(L,t_1)E(L,t_2)E^*(L,t_2)\rangle_0 - \left[\int_{-\infty}^{t} dt_1 \,\langle E(L,t_1)E^*(L,t_1)\rangle_0\right]^2$$
$$= \frac{1}{(2\pi)^2}\int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2 \iiiint d\omega_1 d\omega_2 d\omega_3 d\omega_4 \times e^{-i(\omega_1 t_1-\omega_2 t_1)} e^{-i(\omega_3 t_2-\omega_4 t_2)} \left\langle\Delta^2\sigma_n\right\rangle, \quad (5)$$

which is the background noise coming from the medium and increases continuously with time. As the time integration starts from $-\infty$, in principle, this background noise goes to infinity. Therefore, the background noise needs to be substracted from the signal, which can be achieved by adjusting the reference point of the detector. Therefore, the detected energy variance is given by

$$\left[\left\langle S^2(L,t)\right\rangle - \langle S(L,t)\rangle^2\right] - \left[\left\langle S^2(L,t)\right\rangle_0 - \langle S(L,t)\rangle_0^2\right] = \frac{1}{(2\pi)^2}\int_{-\infty}^{t}\int_{-\infty}^{t} dt_1 dt_2$$
$$\times\iiiint d\omega_1 d\omega_2 d\omega_3 d\omega_4 e^{-i(\omega_1 t_1-\omega_2 t_1)} e^{-i(\omega_3 t_2-\omega_4 t_2)}$$
$$\times\{T(\omega_1,L)T^*(\omega_2,L)T(\omega_3,L)T^*(\omega_4,L)\left\langle\Delta^2\sigma_i\right\rangle + 2T(\omega_1,L)T^*(\omega_4,L)\,\langle E(0,\omega_1)E^*(0,\omega_4)\rangle \times\langle E_n^*(L,\omega_2)E_n(L,\omega_3)\rangle\}. \quad (6)$$

The first term is the modulated original signal noise, the second term is the added noise due to the medium. If the medium is the vacuum, only the first term remains with $T(\omega,z) = e^{i\omega z/c}$ (the photon number variance of the original pulse), which depends on the signal source and is independent of the medium. From the variances, we define the signal-to-noise ratio (SNR) as

$$SNR(L,t) = \frac{[\langle S(L,t)\rangle - \langle S_0(L,t)\rangle]^2}{\left[\langle S^2(L,t)\rangle - \langle S(L,t)\rangle^2\right] - \left[\langle S_0^2(L,t)\rangle - \langle S_0(L,t)\rangle^2\right]}. \quad (7)$$

This SNR is similar to the definition in reference [13]. But in their discussion [13], they do not exclude the background noise, which will be infinite for the second and the fourth terms in equation (10) in reference [13]. We think this background noise should be excluded. It should be

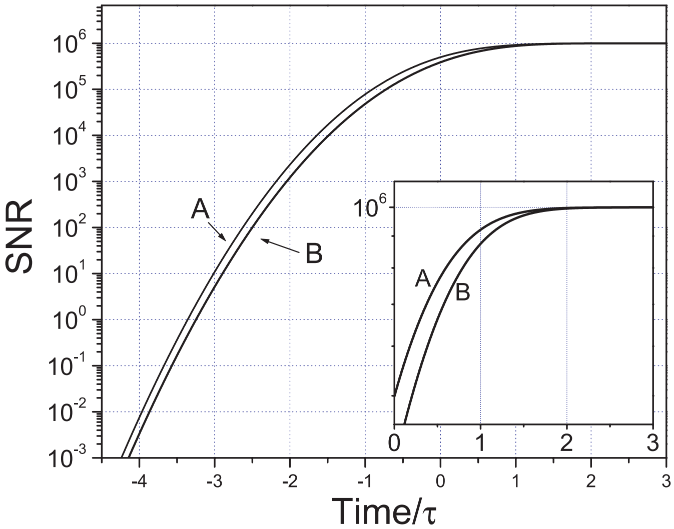

Fig. 1. The SNR of a Gaussian pulse of width $\tau = 1.2$ μs after passing through the medium using a classical treatment (curve B). Curve A is the SNR for the pulse through the same distance vacuum. The medium is the same as in WKD's experiment except for the length of $L = 12$ cm. The peak of the pulse arrives at the incident surface of the medium at time $t = 0$, and emerges from the exit at time $t = -123$ ns. The integration begins from the time -6τ.

noted that our method is a classical method since we do not use quantum mechanics.

The original noise depends only on the source. Different sources give us different original noise. For a laser pulse, even it is fully coherent, it still contains the fluctuation. Here we assume that the original noise in the incident pulse is

$$\langle I(t_1)I(t_2)\rangle - \langle I(t_1)\rangle \langle I(t_2)\rangle = s \langle I(t_1)\rangle \delta (t_2 - t_1), \quad (8)$$

where $I(t) = |E(t)|^2$, and s is a constant. In the frequency domain, the above equation becomes

$$\left\langle \Delta^2 \sigma_i \right\rangle = s \langle E(0,\omega_1)E^*(0,\omega_4)\rangle \delta (\omega_2 - \omega_3). \quad (9)$$

For a Gaussian pulse, $E(0,\omega)$ is also Gaussian distribution over frequency with $|E(0,\omega)|^2$ being the intensity in the mode of frequency ω. For a noiseless pulse, $s = 0$. when $s = 1$, it is equivalent to a coherent-state pulse. The SNR will reach its maximum if we take $E_n(L,\omega) = 0$. Here we would like to emphasize that the dependence of $T(\omega, L)$ on the frequency is exact. In Figure 1, we plot the SNR of a Gaussian pulse after passing through the medium (curve B), and the SNR of the same pulse passing through the same distance vacuum (curve A). In our calculation, the medium is the same as used in the WKD experiment [3], except for the length of $L = 12$ cm. From the figure, we find that curve B is always below curve A, which means that the signal through the medium cannot be received by the detector earlier than that through the vacuum. When the pulse completely passes through the medium, the SNR of the pulse passing the medium approaches the SNR for passing through the vacuum. In Figure 1, $s = 1$ is taken. If s is not equal to one, the scale of the SNR is multiplied by $1/s$.

Note that if we take the approximation $T(\omega, L) \approx T(\omega_0, L)$ in the first term of equation (6) and neglect the high order dependence of the wave number on frequency, we will get the SNR propagated at the speed of v_g in the classical treatment. Here we would like to emphasize that any approximation on the medium might break the Kramers-Kronig relation, which is crucial in the consideration whether the signal velocity is larger than c or not. It will break the causality if we make any approximation in the transmission $T(\omega, L)$.

In conclusion, by using classical theory rather than quantum mechanics, we have shown that the SNR of a pulse passing through an anomalous gain dispersion medium is always smaller than that passing through the same distance vacuum. Thus the signal velocity in the anomalous medium is never faster than c. Although our result is similar to the reference [13], we consider only the classical noise not including the quantum noise. Our calculations reproduce the essential result as in [13], that there is no true superluminality. It should be emphasized that in our calculation, we have not taken the approximations, $|T(\omega, L)| \approx |T(\omega = 0, L)|$ and not neglected the high order dependence of the wave number on frequency. Any approximation in the dispersion relation will lead to an incorrect result even if the approximation is minimal. In fact, as many authors [19,21–24] stated, although the group velocity can be larger than c or even become negative, no actual energy transport occurs faster than c. Therefore, only by using classical theory can we conclude that the signal velocity is less than c.

This work was supported by RGC from HK Government, FRG from HKBU, and the National Natural Science Foundation of China (60268001).

References

1. L. Brillouin, *Wave Propagation and Group Velocity* (Academic, New York, 1960)
2. C.G.B. Garrett, D.E. McCumber, Phys. Rev. A **1**, 305 (1970); S. Chu, S. Wong, Phys. Rev. Lett. **48**, 738 (1982)
3. L.J. Wang, A. Kuzmich, A. Dogariu, Nature **406**, 277 (2000)
4. R.Y. Chiao, A.M. Steinberg, "Tunneling Time and Superluminality", *Progress in Optics*, edited by E. Wolf (Elsevier, 1997), Vol. 43, p. 345
5. R.W. Boyd, D.J. Gauthier, ""Slow" and "Fast" Light", *Progress in Optics*, edited by E. Wolf (Elsevier, 2002), Vol. 43, p. 497
6. G. Nimtz, W. Heitmann, Prog. Quant. Electro. **21**, 81 (1997)
7. G. Nimtz, Prog. Quant. Electro. **27**, 417 (2003)
8. G. Nimtz, IEEE J. Sel. Top. Quant. Electron. **9**, 79 (2003)
9. R.W. Ziolkowski, Phys. Rev. E **63**, 046604 (2001)
10. M.W. Mitchell, R.Y. Chiao, Phys. Lett. A **230**, 133 (1997)

11. M.D. Stenner, D.J. Gauthier, M.A. Neifeld, Nature **425**, 695 (2003)
12. H. Cao, A. Dogariu, L.J. Wang, IEEE J. Sel. Top. Quant. Electron. **9**, 52 (2003)
13. A. Kuzmich, A. Dogariu, L.J. Wang, P.W. Milonni, R.Y. Chiao, Phys. Rev. Lett. **86**, 3925 (2001)
14. K. Wynne, Opt. Commun. **209**, 85 (2002)
15. Y. Aharonov, B. Reznik, A. Stern, Phys. Rev. Lett. **81**, 2190 (1998)
16. B. Segev, P.W. Milonni, J.F. Babb, R.Y. Chiao, Phys. Rev. A **62**, 022114 (2000)
17. J. Peatross, S.A. Glasgow, M. Ware, Phys. Rev. Lett. **84**, 2370 (2000)
18. A. Dogariu, A. Kuzmich, H. Cao, L.J. Wang, Opt. Expr. **8**, 344 (2001)
19. N.H. Liu, S.Y. Zhu, H. Chen, X. Wu, Phys. Rev. E **65**, 046607 (2002)
20. L.G. Wang, N.H. Liu, Q. Lin, S.Y. Zhu, Europhys. Lett. **60**, 834 (2002)
21. L.G. Wang, N.H. Liu, Q. Lin, S.Y. Zhu, Phys. Rev. E **68**, 066606 (2003)
22. D. Diener, Phys. Lett. A **235**, 118 (1997)
23. M. Ware, S.A. Glasgow, J. Peatross, Opt. Expr. **9**, 506 (2001)
24. H.G. Winful, Phys. Rev. E **68**, 016615 (2003)